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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Calculus, 2009

90635 Differentiate functions and use derivatives to solve problems

Credits: Six
2.00 pm Thursday 26 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show any derivatives that you need to find when solving the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Differentiate functions and use derivatives to solve problems. ☐	Demonstrate knowledge of advanced concepts and techniques of differentiation and solve differentiation problems. ☐	Solve more complex differentiation problem(s). ☐
Overall Level of Performance ☐		

You are advised to spend 50 minutes answering the questions in this booklet.

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QUESTION ONE

(a) Differentiate $y = (\sin x + 1)^4$.

You do not need to simplify your answer.

(b) Find the equation of the tangent to $y = \dfrac{1}{x+1}$ at $x = 1$.

Give any derivatives you need to find when solving this problem.

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(c) Find $\frac{dy}{dx}$ if $4xy^2 - 3y = 4x^2$.

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(d) The function f is given by:

$f(x) = e^{\frac{x}{1+kx}}$, where k is a positive constant.

Prove that $f(x)$ has a point of inflection at $x = \frac{1}{2k^2} - \frac{1}{k}$.

Give any derivatives you need to find when solving this problem.

QUESTION TWO

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(a) Differentiate $y = \ln(4x^2 + 3)$.

You do not need to simplify your answer.

(b) Find the x coordinates of all the stationary points on the curve $y = x^2e^x$.

Give any derivatives you need to find when solving this problem.

(c) An isosceles triangle is shown below.

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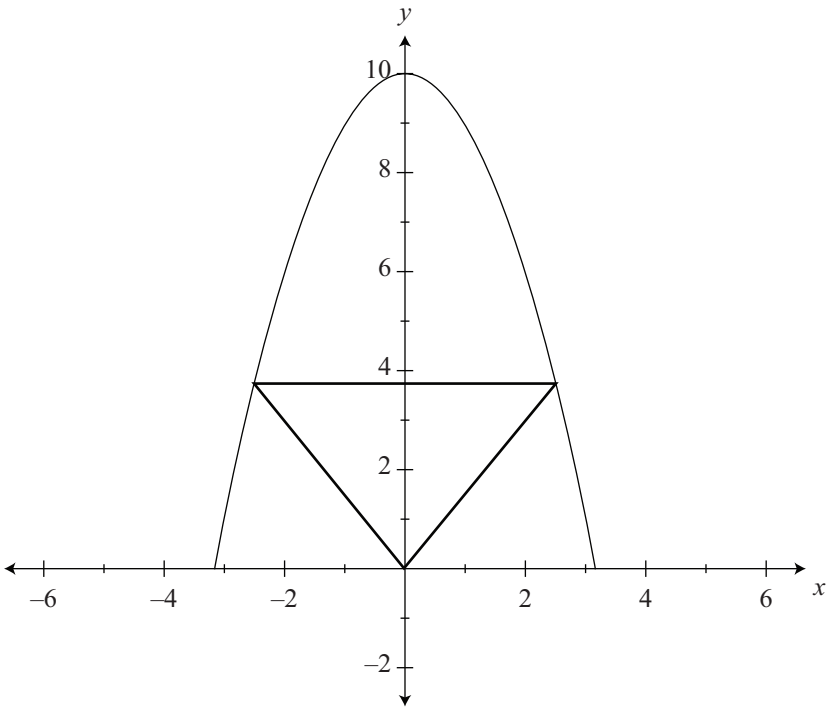

One vertex of the triangle is at (0,0).
The other two vertices are on the curve $y = 10 - x^2$, both vertices having the same y coordinate.

Find the maximum possible area, A, of such a triangle.

You may assume that $\frac{d^2A}{dx^2} < 0$.

Give any derivatives you need to find when solving this problem.

(d) Find all the values of t at which the curve defined parametrically by

$$x = 2t^3 - 15t^2 + 24t + 7$$

and $y = 6e^{t^2 - 4}$

has a tangent that is vertical.

Give any derivatives you need to find when solving this problem.

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Question Two (e) is on the following page.

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(e) The pressure, P, on a scuba diver is a function of the diver's depth.

$P = kd + 1$, where P is the pressure on the diver in atmospheres
d is the depth below the surface in metres
and k is a constant.

The pressure is 1 atmosphere at the surface, and 4 atmospheres at a depth of 30 metres.

A diver does not descend at a constant rate. Her rate of descent depends on her depth and her experience.

A diver's depth at time t is given by the formula

$d = 1 + 10 \ln(nt)$ where d is the depth below the surface in metres
n is the number of dives she has completed in the past 12 months
and t is the time since she left the surface.

This diver is descending.

In terms of n, find the rate of change, with respect to time, of the pressure when the diver is at a depth of 25 metres.

Give any derivatives you need to find when solving this problem.

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Extra paper for continuation of answers if required.
Clearly number the question.

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Question number

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